Exercise: Solve the RG equation

$$\frac{d}{d\ln\mu} \tilde{C}_V(Q^2,\mu^2) = \left[-2C_F\gamma_{cusp}\ln\frac{\mu}{Q} + \gamma_V\right]\tilde{C}_V(Q^2,\mu^2)$$

$$\frac{d\alpha}{d\ln\mu} = \beta(\alpha) \qquad \ln\left(\frac{\mu}{Q}\right) = \int_{\alpha(Q)}^{\alpha(\mu)} \frac{d\alpha}{\beta(\alpha)}$$

$$\frac{dC_V}{C_V} = \frac{d\alpha}{\beta(\alpha)}\left[-2C_F\gamma_{cusp}\int_{\alpha(Q)}^{\alpha}\frac{d\alpha'}{\beta(\alpha')} + \gamma_V\right]$$

$$\ln\left(\frac{C_V(Q^2,\mu)}{C_V(Q^2,\mu_h)}\right) = \int_{\alpha(\mu_h)}^{\alpha(\mu)}\frac{d\alpha}{\beta(\alpha)}\left[-2C_F\gamma_{cusp}\int_{\alpha(Q)}^{\alpha}\frac{d\alpha'}{\beta(\alpha')} + \gamma_V\right]$$

write $\displaystyle\int_{\alpha(Q)}^{\alpha}\frac{d\alpha'}{\beta(\alpha')} = \int_{\alpha(\mu_h)}^{\alpha}\frac{d\alpha'}{\beta(\alpha')} - \ln\left(\frac{Q}{\mu_h}\right)$

$$C_V(Q^2,\mu) = C_V(Q^2,\mu_h)\exp\Big[2C_F S(\mu_h,\mu) - A_{\gamma_V}(\mu_h,\mu) - C_F\ln\left(\frac{Q^2}{\mu_h^2}\right)A_{\gamma_{cusp}}(\mu_h,\mu)\Big] \checkmark$$