Soft-Collinear Effective Theory

Part IV: Resummation by RG evolution

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Outline

Part I: The strategy of regions

Part II: Scalar SCET

Part III: Generalization to QCD

Part IV: Resummation by RG evolution

Part V: IR divergences of gauge theory amplitudes

Resummation by renormalization group (RG) evolution

We now discuss renormalization and RG evolution in the effective theory.

It would be nice to resum logarithmically enhanced contributions to a *physical* cross section. SCET has been used to resum log's in DIS, Drell-Yan, Higgs production, event shapes, ...

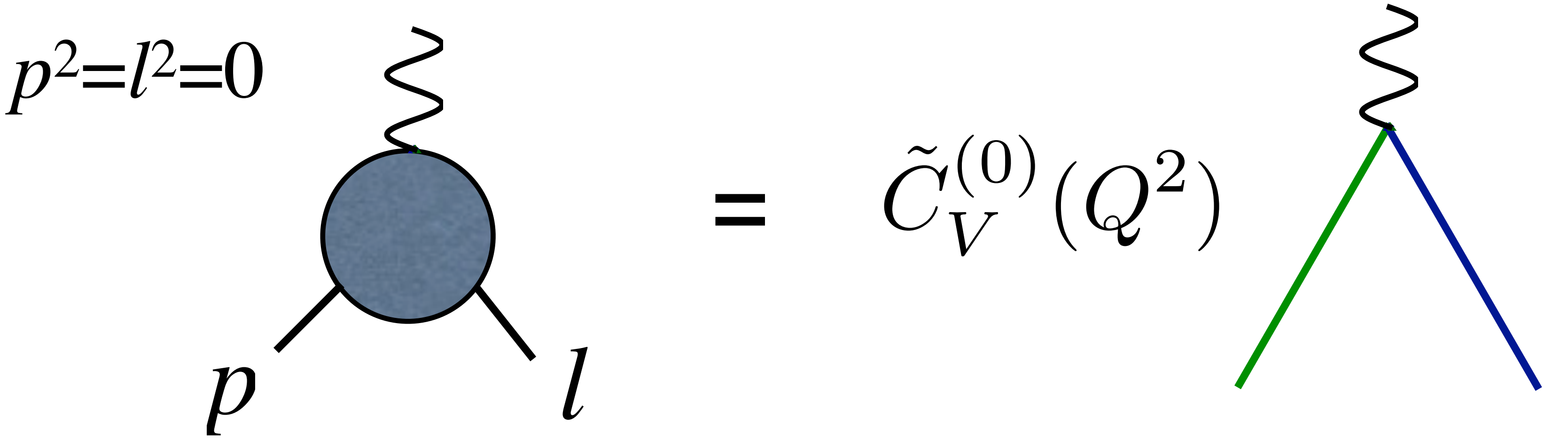
For simplicity, we'll instead just discuss the Sudakov form factor. In this case, we have just derived the necessary factorization theorem. Furthermore it also serves as an introduction to the discussion of *n*-point amplitudes in the last lecture.

Matching

The Fourier transform of the matching coefficient of the current operator

$$\psi(x)\gamma^{\mu}\psi(x) \to \int ds dt\, C_V(s,t)\, \chi_{c1}(x+s\bar{n})\, \gamma_{\perp}^{\mu}\, \chi_{c2}(x+tn)$$

is obtained by evaluating the on-shell form factor.



This gives the *bare* Wilson coefficient $\tilde{C}_V^{(0)}(Q^2)$.

Renormalization

At one loop, one finds (3-loop is known!)

$$\widetilde{C}_V^{(0)}(Q^2) = 1 + \frac{\alpha_s^{(0)}}{4\pi} C_F \left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \frac{\pi^2}{6} \right) (Q^2)^{-\epsilon}$$

Let us define a renormalized Wilson coefficient by absorbing the divergences into a *Z*-factor:

$$\widetilde{C}_V(Q^2, \mu^2) = \lim_{\epsilon \to 0} Z^{-1}(Q^2, \mu^2)\, \widetilde{C}_V^{(0)}(Q^2)$$

where $\alpha_s^{(0)} = Z_\alpha\, \mu^{2\epsilon} \alpha_s(\mu)$ and

$$Z(Q^2, \mu^2) = 1 + \frac{\alpha_s}{4\pi} C_F \left[-\frac{2}{\epsilon^2} - \frac{2}{\epsilon} \ln \frac{Q^2}{\mu^2} - \frac{3}{\epsilon} \right]$$

RG equation

The renormalized Wilson coefficient

$$\widetilde{C}_V(Q^2,\mu^2) = 1 + \frac{\alpha_s(\mu)}{4\pi} C_F \left(-\ln^2 \frac{Q^2}{\mu^2} + 3\ln \frac{Q^2}{\mu^2} + \frac{\pi^2}{6} - 8 \right)$$

fulfills the renormalization group equation:

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu} C_V(Q^2,\mu^2) = \left[C_F\, \gamma_{\mathrm{cusp}}(\alpha_s)\, \ln \frac{Q^2}{\mu^2} + \gamma_V(\alpha_s) \right] C_V(Q^2,\mu^2)$$

At one loop:

$$\gamma_{\mathrm{cusp}}(\alpha_s) = 4\, \frac{\alpha_s(\mu)}{4\pi} \quad \text{and} \quad \gamma_V(\alpha_s) = -6\, C_F\, \frac{\alpha_s(\mu)}{4\pi}$$

name will be explained later

Aside: Three-loop anomalous dimension

Divergencies of the on-shell form factor are known to 3-loops , (Moch, Vermaseren Vogt '05). Since last year, also the finite pieces are known at this accuracy. Using these results, one can extract the anomalous dimension to three loops:

$$
\begin{aligned}
\gamma_0^V &= -6C_F = -8\,,\\
\gamma_1^V &= C_F^2\left(-3+4\pi^2-48\zeta_3\right)+C_FC_A\left(-\frac{961}{27}-\frac{11\pi^2}{3}+52\zeta_3\right)+C_FT_Fn_f\left(\frac{260}{27}+\frac{4\pi^2}{3}\right)\\
&\approx 1.1419\,,\\
\gamma_2^V &= C_F^3\left(-29-6\pi^2-\frac{16\pi^4}{5}-136\zeta_3+\frac{32\pi^2}{3}\,\zeta_3+480\zeta_5\right)\\
&\quad+C_F^2C_A\left(-\frac{151}{2}+\frac{410\pi^2}{9}+\frac{494\pi^4}{135}-\frac{1688}{3}\,\zeta_3-\frac{16\pi^2}{3}\,\zeta_3-240\zeta_5\right)\\
&\quad+C_FC_A^2\left(-\frac{139345}{1458}-\frac{7163\pi^2}{243}-\frac{83\pi^4}{45}+\frac{7052}{9}\,\zeta_3-\frac{88\pi^2}{9}\,\zeta_3-272\zeta_5\right)\\
&\quad+C_F^2T_Fn_f\left(\frac{5906}{27}-\frac{52\pi^2}{9}-\frac{56\pi^4}{27}+\frac{1024}{9}\,\zeta_3\right)\\
&\quad+C_FC_AT_Fn_f\left(-\frac{34636}{729}+\frac{5188\pi^2}{243}+\frac{44\pi^4}{45}-\frac{3856}{27}\,\zeta_3\right)\\
&\quad+C_FT_F^2n_f^2\left(\frac{19336}{729}-\frac{80\pi^2}{27}-\frac{64}{27}\,\zeta_3\right)\approx -249.388\,.
\end{aligned}
$$

What is special, is that the RG equation

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\,\widetilde{C}_V(Q^2,\mu^2) = \left[C_F\,\gamma_{\mathrm{cusp}}(\alpha_s)\,\ln\frac{Q^2}{\mu^2} + \gamma_V(\alpha_s)\right]\widetilde{C}_V(Q^2,\mu^2)$$

contains explicit logarithmic dependence on μ. This is characteristic for problems with Sudakov double logarithms.

The solution of this equation (*exercise*) sums the logarithmic terms to all orders. To obtain the solution, use that

$$\frac{\mathrm{d}\alpha_s}{d\ln\mu} = \beta(\alpha_s)$$

Solution

$$\widetilde{C}_V(Q^2,\mu) = U(\mu_h,\mu)\,\widetilde{C}_V(Q^2,\mu_h)$$

Evolution factor

$$U(\mu_h,\mu) = \exp\left[2C_F S(\mu_h,\mu) - A_{\gamma_V}(\mu_h,\mu)\right] \left(\frac{Q^2}{\mu_h^2}\right)^{-C_F\,A_{\gamma_\text{cusp}}(\mu_h,\mu)}$$

with

$$S(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} \mathrm{d}\alpha \frac{\gamma_\text{cusp}(\alpha)}{\beta(\alpha)} \int_{\alpha_s(\nu)}^{\alpha} \frac{\mathrm{d}\alpha'}{\beta(\alpha')}$$

(double log's)

$$A_\gamma(\nu,\mu) = -\int_{\alpha_s(\nu)}^{\alpha_s(\mu)} d\alpha \frac{\gamma(\alpha)}{\beta(\alpha)}$$

(single log's)

The explicit solution is obtained by plugging in the perturbative expansion for the β-function and the anomalous dimensions

$$\beta(\alpha_s) = -2\alpha_s \left[\beta_0 \frac{\alpha_s}{4\pi} + \beta_1 \left(\frac{\alpha_s}{4\pi}\right)^2 + \dots\right]$$

$$\gamma_{\text{cusp}}(\alpha_s) = \Gamma_0 \frac{\alpha_s}{4\pi} + \Gamma_1 \left(\frac{\alpha_s}{4\pi}\right)^2 + \dots$$

and performing the integrals $r = \alpha_s(\mu)/\alpha_s(\nu)$

$$S(\nu,\mu) = \frac{\Gamma_0}{4\beta_0^2} \left\{ \frac{4\pi}{\alpha_s(\nu)} \left(1 - \frac{1}{r} - \ln r\right) + \left(\frac{\Gamma_1}{\Gamma_0} - \frac{\beta_1}{\beta_0}\right)(1 - r + \ln r) + \frac{\beta_1}{2\beta_0} \ln^2 r \right\}$$

$$A_\gamma(\nu,\mu) = \frac{\gamma_0}{2\beta_0} \ln \frac{\alpha_s(\mu)}{\alpha_s(\nu)}$$

Red part corresponds to “leading-log accuracy”.

The perturbation expansion for the fixed order result for C_V

$$\widetilde{C}_V(Q^2,\mu^2) = 1 + \frac{\alpha_s(\mu)}{4\pi} C_F \left(-\ln^2 \frac{Q^2}{\mu^2} + 3 \ln \frac{Q^2}{\mu^2} + \frac{\pi^2}{6} - 8 \right)$$

breaks down for $\mu \ll Q$ or $\mu \gg Q$ because the logarithms become large.

In contrast, for $\mu_h \approx Q$ the RG improved result

$$\widetilde{C}_V(Q^2,\mu) = U(\mu_h,\mu)\, \widetilde{C}_V(Q^2,\mu_h)$$

is valid for any scale μ for which α_s is perturbative.

Why resummation?

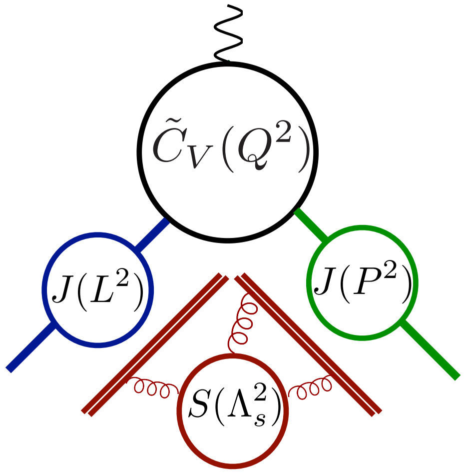
In problems with widely separated physical scales $\Lambda_1 \gg \Lambda_2$ fixed order perturbation theory is not appropriate

- Large logarithms $\alpha_s^n \,\mathrm{Log}^n(\Lambda_1/\Lambda_2)$ and Sudakov logarithms $\alpha_s^n \,\mathrm{Log}^{2n}(\Lambda_1/\Lambda_2)$.
- Scale in coupling? $\alpha_s(\Lambda_1)$ or $\alpha_s(\Lambda_2)$?

Standard solution

- Use effective theories to separate the effects associated with different scales.
- RG evolution in the effective theory resums large log’s.

Factorization



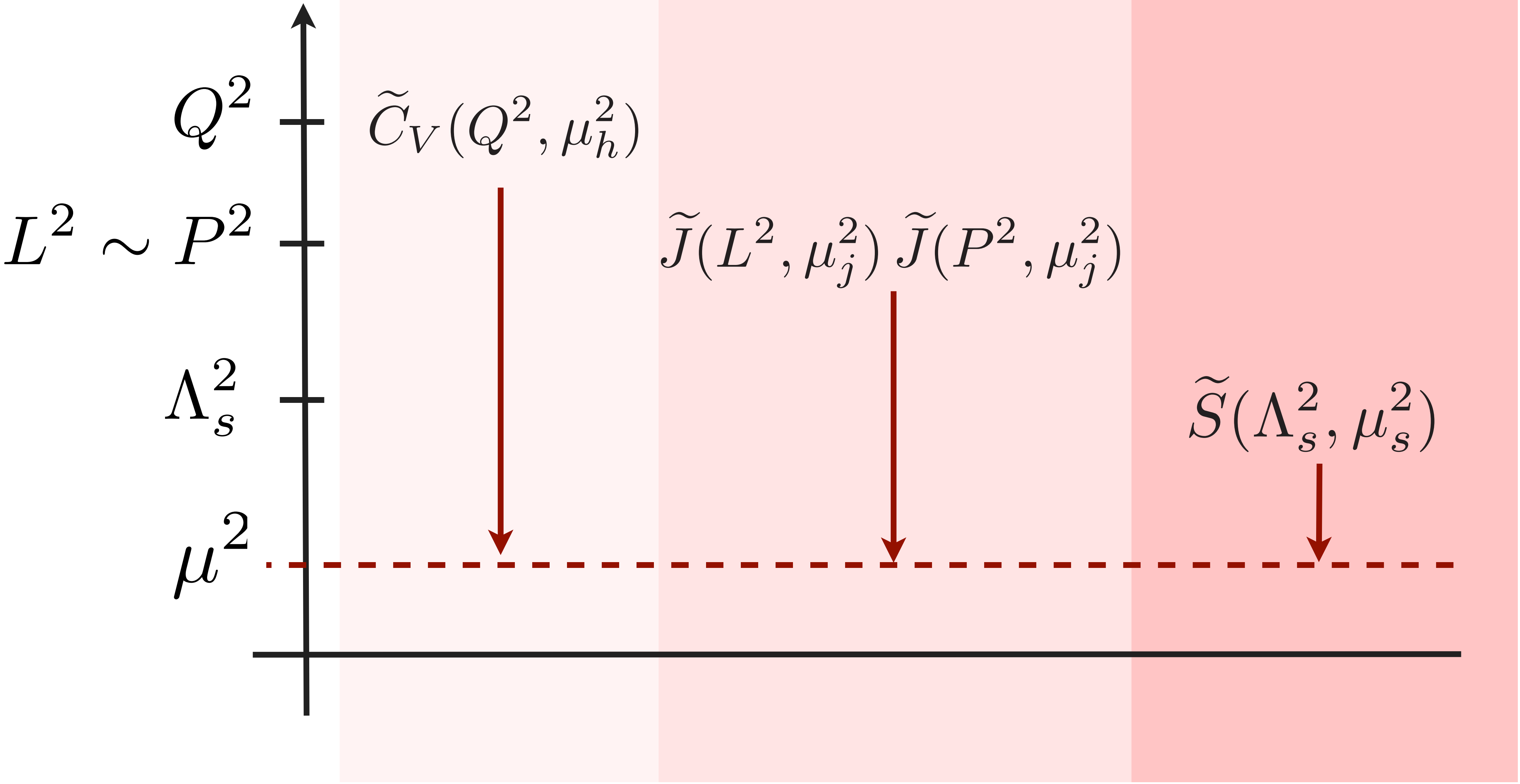
We have integrated out the hard contribution and absorbed it into the Wilson coefficient $\widetilde{C}_V(Q^2, \mu^2)$.

The decoupling further factorized the soft and collinear interactions, so that our matrix element factorizes into collinear functions $\widetilde{J}(L^2, \mu^2)$ and $\widetilde{J}(P^2, \mu^2)$ times a soft function $\widetilde{S}(\Lambda_s^2, \mu^2)$.

$$F(Q^2, L^2, P^2) = \widetilde{C}_V(Q^2, \mu^2)\, \widetilde{J}(L^2, \mu^2)\, \widetilde{J}(P^2, \mu^2)\, \widetilde{S}(\Lambda_s^2, \mu^2)$$

Each function fulfills a RG equation of the same structure as the one for $\widetilde{C}_V(Q^2, \mu^2)$.

Evaluate each part at its characteristic scale, evolve to common reference scale μ



Each contribution is evaluated at its natural scale. No large perturbative logarithms.

Factorization constraint on the anomalous dimensions

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu}\ln\left[\widetilde{C}_V(Q^2,\mu^2)\,\widetilde{J}(L^2,\mu^2)\,\widetilde{J}(P^2,\mu^2)\,\widetilde{S}(\Lambda_s^2,\mu^2)\right]=0$$

$$=\Gamma_{\mathrm{hard}}+\Gamma_{\mathrm{jet}}+\Gamma_{\mathrm{soft}}$$

$$=+\,C_F\gamma_{\mathrm{cusp}}\ln\frac{Q^2}{\mu^2}+\gamma_V$$

$$+\,C_F\gamma_{\mathrm{cusp}}\left(\ln\frac{L^2}{\mu^2}+\ln\frac{P^2}{\mu^2}\right)+2\gamma_J$$

$$-\,C_F\,\gamma_{\mathrm{cusp}}\ln\frac{\mu^2}{\Lambda_s^2}+\gamma_s$$

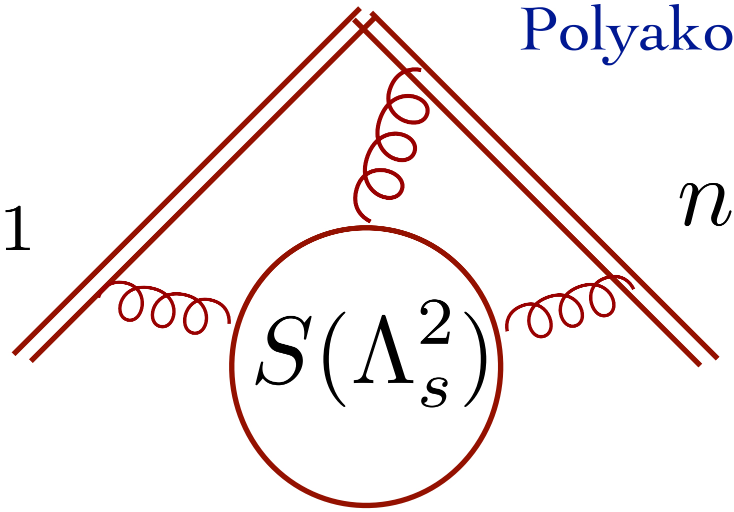
$$\Lambda_s^2=\frac{L^2P^2}{Q^2}$$

For this cancellation to work, it is crucial that the scale dependence is logarithmic, with a universal coefficient.

Cusp anomalous dimension

Wilson lines with cusps require renormalization and the anomalous dimension is proportional to the cusp angle.

Polyakov 1980; Brandt, Neri, Sato 1981



The cusp angle $\cosh\beta_{12} = \frac{n_1 \cdot n_2}{\sqrt{n_1^2\, n_2^2}}$ diverges for light-like Wilson lines. Anomalous dimension has the form

$$\Gamma(\beta_{12}) \overset{n_{1,2}^2 \to 0}{\longrightarrow} \Gamma^i_{\text{cusp}}(\alpha_s)\, \ln\frac{\mu^2}{\Lambda_s^2} + \ldots$$

Korchemskaya, Korchemsky 1992