Problem Set 0.

Reviewing basic quantum mechanics

Problem 0.1

Find the energy-dependent reflection and transmission coefficients for the scattering of a one-dimensional particle on a δ-function potential:

$$H = \frac{p^2}{2m} + V\delta(x)\,. \tag{1}$$

Consider both cases of positive V (potential barrier) and negative V (potential well).

Problem 0.2

(a) Derive the uncertainty relation: Let A, B, and C be Hermitian operators such that $[A, B] = iC$. Then prove that, under certain conditions on the wave function, the following inequality holds:

$$\Delta A\; \Delta B \geq \frac{\langle C\rangle}{2}\,, \tag{2}$$

where

$$\Delta A = \Big(\langle A^2\rangle - \langle A\rangle^2\Big)^{1/2} \tag{3}$$

(and similarly for ΔB), and $\langle\ldots\rangle$ denotes the expectation value in a given quantum state.

(b) Compute the momentum–coordinate uncertainty $\Delta p\; \Delta x$ in the n-th state of the one-dimensional harmonic oscillator

$$H = \frac{p^2}{2m} + \frac{kx^2}{2}\,. \tag{4}$$

Problem 0.3

Find the discrete spectrum of a free one-dimensional particle in the $\cosh^{-2}$ potential well. The Hamiltonian is as follows:

$$H = \frac{p^2}{2m} - \frac{U}{\cosh^2(x/a)}\,. \tag{5}$$

Here p and x are the canonical momentum and coordinate of the paritcle, m, U, and a are the parameters of the Hamiltonian. Hint: look for eigenfunctions of the form $\Psi(x) = \cosh^{-\nu}(x/a)\, p(\sinh(x/a))$, where p is a polynomial.