Quantum Physics III, 2005 − 2006

Problem Set 1.
Quasiclassical energy levels.

Problem 1.1

In the quasiclassical approximation, find the energy spectrum of the one-dimensional "half-oscillator":

$$H = \frac{p^2}{2} + U(x)\,, \tag{1}$$

$$U(x) = \begin{cases} +\infty\,, & x < 0 \\ \omega^2 x^2/2 & x > 0 \end{cases} \tag{2}$$

Problem 1.2

In the quasiclassical approximation, find the discrete energy spectrum of a one-dimensional particle in the potential $-U \cosh^{-2}(x/a)$. Compare with the exact result (see Problem 0.3).

Problem 1.3∗

(a) Consider a particle moving in a three-dimensional central potential $U(r)$ such that $U(r \to 0)$ is finite. In the quasiclassical approximation, find the energy levels at a given angular momentum $l \sim 1$. (Hint: at $r \to 0$, match the quasiclassical wave functions to the Bessel-function solutions!)

(b) Apply the result to the spherically symmetric harmonic oscillator $U(r) = kr^2/2$. Compare with the exact spectrum.