Problem Set 2.

Quasiclassical wave functions.

Problem 2.1

(a) Consider a one-dimensional particle in an eigenstate of a discrete spectrum in a potential well $U(x)$. In the quasiclassical approximation, compute the probability for the particle to be in the classically prohibited region. Is it true that this probability is much smaller than one? [Hint: the integral $\int_0^\infty (Ai(z))^2\, dz$ is just a number. Its approximate value is $\int_0^\infty (Ai(z))^2\, dz \approx 0.06699...$ You don't need to compute it analytically.]

(b) Apply the result to the n-th level of the harmonic oscillator.

Problem 2.2

Consider a one-dimensional particle in an eigenstate of a discrete spectrum in a potential well $U(x)$. In the quasiclassical approximation, what is the expectation value $\langle F(x) \rangle$ of an observable depending only on the coordinate x? Consider an example of the harmonic oscillator, and compute in the quasiclassical approximation $\langle x^2 \rangle$ and $\langle x^4 \rangle$ in the n-th eigenstate. Compare with the exact result.