## Problem Set 5.

Scattering amplitude and cross-section. Born approximation.

**Problem 5.1**

In the Born approximation, find the scattering amplitude and the total scattering cross-section for the following centrally symmetric potentials:

(a) $U(r) = U_0 e^{-r/R}$;

(b) $U(r) = U_0 e^{-r^2/R^2}$.

(c) $U(r) = \alpha/r^2$.

Specify the limits of applicability of the Born approximation.

**Problem 5.2**

In the Born approximation, the forward-scattering amplitude (the scattering angle $\theta = 0$) is real, and therefore it appears to be in contradiction with the *optical theorem*. Resolve the contradiction. Compute the second-order perturbative correction to the scattering aplitude and verify the optical theorem to the second order in the perturbation theory.

**Problem 5.3**

Calculate the phase shifts $\delta_l(k)$ in the scattering potential $U(r) = \alpha/r^2$. Using this result, calculate the scattering amplitude $f(k, \theta)$ from the relation

$$f(k,\theta) = \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)[e^{2i\delta_l} - 1]P_l(\cos\theta) \tag{1}$$

in the following limits:

(a) $m\alpha/\hbar^2 \ll 1$ for arbitrary scattering angle;

(b) $m\alpha/\hbar^2 \gtrsim 1$ for small scattering angles;

(c) $m\alpha/\hbar^2 \gg 1$ for backward scattering ($\theta = \pi$).

Compare to the Born-approximation results (Problem 5.1).