Quantum Physics IV, 2004 – 2005

Final exam.

The exam will consist of **(1)** one theory question (randomly chosen from the list below), **(2)** one problem (distributed at the exam, not known in advance), and **(3)** possible additional questions on the subject of the course. The problem will be similar to those from the exercises, but easier than the average level of the exercise problem sets. After the random selection of the theory topic and the problem, you will have 15-30 minutes to solve the problem and to prepare the presentation of the theory question. You should aim at a 5-10-minute presentation of the theory question and a 5-10-minute presentation of the problem. During your preparation, you may use any material you wish, such as books, lecture notes, etc., but you may not use help of others.

List of theory questions.

1. Propagator of the harmonic oscillator.
Explain why the propagator has the form

$$K(t_f, x_f; t_i, x_i) = J(t_f - t_i) e^{\frac{i}{\hbar} S_{\rm cl}} ,$$

where $J((t_f - t_i)$ depends only on the time difference, and $S_{\rm cl}$ is the action along the classical trajectory.

2. Integration measure.
For one-dimensional particle with the Hamiltonian

$$H = \frac{p}{2m^2} + U(x)$$

define the integration measures in the functional integrals in the coordinate space

$$K(t_f, x_f; t_i, x_i) = \int \mathcal{D}x(t)\, e^{\frac{i}{\hbar} S[x(t)]}$$

and in the phase space

$$K(t_f, x_f; t_i, x_i) = \int \mathcal{D}x(t)\, \mathcal{D}p(t)\, e^{\frac{i}{\hbar} S[x(t),p(t)]} .$$

3. Fluctuation determinant of the instanton.
For the double-well problem

$$H = \frac{p}{2m^2} + U(x) ,$$

where the potential $U(x)$ has two degenerate classical minima, define the fluctuation determinant of the instanton.

4. Gelfand-Yaglom determinant formula.
Explain the Gelfand-Yaglom formula

$$\det \left[-\frac{d^2}{dt^2} + U(t) \right] = \frac{1}{\tau} \varphi(\tau), \qquad \varphi(0) = 0, \quad \dot{\varphi}(0) = 1 .$$

What is the normalization of the determinant on the left-hand side?

5. Landau levels.
For a two-dimensional charged particle of mass m in a uniform magnetic field B, what is the density of states in each of the Landau levels?

6. Path integral for spins.
Explain the definition of the two terms in the action for the spin s:

$$S[\mathbf{n}(t)] = s \int_{\Sigma} d^2\Omega - \int H(\mathbf{n}(t))\, dt\,.$$

In a uniform field, derive the classical equation of motion from the variational principle.

7. Path integral for bosons.
In terms of coherent states, the action for the harmonic oscillator is

$$S = \int dt\, a^*(t) \left(i\frac{d}{dt} - \Omega \right) a(t)$$

Make the substitution

$$a = \frac{q+ip}{\sqrt{2}}, \qquad a^* = \frac{q-ip}{\sqrt{2}}$$

and compare the result to the action of the harmonic oscillator in the phase space

$$S(p,q) = \int p\, dq - \int H(p,q)\, dt\,.$$

8. Coherent states for fermions.
Explain the construction of coherent states for fermions $|\eta\rangle$ and $\langle\eta|$. Show that (with a suitable normalization of the coherent states),

$$\int d\bar{\eta}\, d\eta\, |\eta\rangle\langle\eta| = \mathbf{1}\,.$$