Problem Set 1.

Propagators in quantum mechanics

Problem 1.1

Consider two one-dimensional systems:

(1) free particle,

$$H = \frac{p^2}{2m} \tag{1}$$

(2) harmonic oscillator,

$$H = \frac{p^2}{2m} + \frac{m\omega^2}{2}x^2 \tag{2}$$

For both systems, find the propagators $K(x, t; x', t')$ (in real-space representation, i.e., as functions of x and t). Show that they have the same asymptotic behavior as $(t - t') \to 0$. Will it be true for any potential?

Problem 1.2

For a stationary system, the propagator $K(x, t; x', t')$ depends only on the time difference $t - t'$. Let $K(x, x', \omega)$ denote the Fourier transform of $K(x, t; x', t')$ in $t - t'$. Express the density of states (number of energy levels in a given energy interval) in terms of $K(x, x', \omega)$. Check that your propagators from the Problem 1.1 reproduce the known answers for the density of states.

Problem 1.3

Verify that the propagators $K(x, x', t, t')$ which you have obtained in Problem 1.1 obey the "superposition" rule: for any three time moments $t_3 > t_2 > t_1$

$$K(x_3, t_3; x_1, t_1) = \int dx_2\, K(x_3, t_3; x_2, t_2)\, K(x_2, t_2; x_1, t_1) \tag{3}$$

Problem 1.4

Consider an one-dimensional particle in the attractive δ-potential:

$$H = \frac{p^2}{2m} - \alpha\delta(x) \tag{4}$$

Treating the potential as a perturbation, write the perturbation series for the Green's function $G(p, \omega)$. Sum the perturbation series and show that the Green's function acquires a new pole at ω corresponding to the energy of the bound state.

Can you do the same calculation in two and three dimensions?